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Influence of Problem Solving Approach on Secondary School Students' Mathematics Achievement in Commercial Arithmetics in Kenya

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Abstract: **In Kenya, the fundamental challenge facing learning of mathematics in secondary schools is how to enhance students' conceptual understanding associated with the teaching/learning process. Based on this challenge, the present study investigated the influence of using Problem Solving Approach on secondary school students' mathematics achievement in Commercial Arithmetics in Kenya. The purpose of this study was to determine whether the use of Problem Solving Approach has any influence on students' mathematics achievement in Commercial Arithmetics. Students from one hundred and nine schools from Vihiga County formed the population of the study. Stratified random sampling was used to select twelve schools from the 109 schools. A total of 1459 Form Three students were selected from the twelve schools that participated in the study. The respondents were from national, county and district schools. The Solomon Four-Group design was used in the study. The respondents were assigned in their intact classes to four groups namely; experimental groups 1 and 3, and control groups 2 and 4. All the groups were taught the same content of the topic Commercial Arithmetics. However, groups 1 and 3 were taught using Problem Solving Approach while groups 2 and 4 were taught by conventional methods. Groups 1 and 2 were pre-tested prior to the implementation of the Problem Solving Approach treatment. Mathematics Achievement Test 1 and Mathematics Achievement Test 2 were used to collect data. The instruments' validity was determined by the researcher, a panel of mathematics educators from the Department of Science and Mathematics Education at Masinde Muliro University of Science and Technology and experienced secondary school mathematics teachers. Reliability coefficients of 0.795 and 0.872 were obtained for Mathematics Achievement Test 1 and Mathematics Achievement Test 2 respectively using Cronbach's Coefficient alpha formula. After the treatment, all the four groups were post-tested. Significance of differences between the means of the experimental and control groups on the variables of pre-test and post-test were measured by the t-test and Analysis of Variance. The results show that increased students' learning occurred among students where Problem Solving Approach was used. The researcher concluded that Problem Solving Approach is an effective teaching approach. It helpful in enhancing the teaching and learning of mathematics, facilitated in making the subject easily understandable to students and consequently their achievement in the subject. Therefore, mathematics educators should encourage mathematics teachers to use it and teacher educators to make it part of the teacher-training curriculum.**

Keywords: Problem Solving Approach, Secondary School, Students' Mathematics Achievement, Kenya.

1. INTRODUCTION

Mathematics is one of the core subjects in the Kenya secondary school curriculum. It is an examinable subject for all students (Kenya Institute of Curriculum Development [KICD], 2006). Much importance is currently attached to it by the society. As a tool, it finds its application in daily lives at home, in the office and in scientific and technological fields.

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Despite its importance, students have consistently performed poorly in the subject. This is evident from the Kenya Certificate of Secondary Education (KCSE) examination results. The years 2006, 2007, 2008 and 2009 recorded low mean scores of 38.08, 39.46, 42.59 and 42.26 respectively (KNEC, 2010). The mean score figures indicate that there was a slight decline in the overall mean score in the year 2009 compared to the previous year. However, the general performance in the subject is poor as depicted by the low mean scores. This poor performance was attributed to poor teaching and/or learning strategies (KNEC, 2007-2009).

In the recent past, teaching and learning practices have undergone changes of revolutionary proportions; changes underpinned by shifts in psychological and pedagogical theory in teaching and learning process. The new developments advocates for new approaches to mathematics teaching and learning, not only in secondary schools but also in teacher education (Okigbo & Osuafor, 2008). Research findings on learning and memory show that for learning to be effective, the learner should be actively involved in the learning process (Lambros, 2002). Piaget believed that there is no true learning unless the students mentally act on information and in the process, assimilate or accommodate what they encounter in their environment (Trowbridge & Bybee, 2004). Unless this assimilation occurs, teachers and students are involved in pseudo-learning, which is knowledge retained only for short time. Efforts made to translate these new conceptions of learning into classroom practices include development of instructional methods that engage the learner actively in the process of knowledge acquisition. Mathematical problem solving is a teaching approach that is learner-centred. It may improve and motivate students' learning, problem solving skills and broad mathematics knowledge, based on deep understanding and problem solving (Major *et al.*, 2000)

Cognitive psychology research has provided considerable insight into the way the learners acquire and organize knowledge. A growing body of research today points to active learning strategies in which the students listen, talk, write, read and reflect as they become directly involved in the instructional process (Roh, 2003). Constructivist theories of learning which had its roots from cognitive psychology place the learner in an active role of knowledge construction. The learner approaches a domain with some prior knowledge about the subject matter constructed from personal experiences, schooling, and social interactions (Okere, 1996). Concepts change as the learner attempts to connect new information with existing conceptual framework. According to constructivist theories of learning, conceptual change in learners should be facilitated by problem solving activities such as having students actively engaged in processing knowledge; confronting their conceptual framework; confronting defending alternatives perspective; linking new concepts to old; and using strategies that encourages both meta-cognition and higher order thinking (Walker & Lofton, 2003).

Effective strategies designed to promote efficient and meaningful learning rely upon connecting prior knowledge to new concepts (Cook, 2001; Okerere, 2006). The importance of meaningful learning in promoting conceptual understanding that in turn facilitates problem solving was stressed by Bransford and Stein (1984), Eylon and Linn (1988) and Mangle (2008). Research in different areas in mathematics and in other subjects has established the existence of positive relationships between students' meaningful learning approaches and their achievement in mathematics (Wentzel, 2002; Boaler, 2002; Samuelsson, 2008). According to Ramsden (1995) meaningful learners have a deep approach to learning. They tend to build a holistic description of content, reorganise new content by relating it to prior knowledge and/or to personal experiences, are inclined to use evidence, and maintain a critical and a more objective view. Conversely, rote learners have a surface approach to learning; they have a propensity for memorisation of mathematics facts, concepts, principles and strategies and are motivated extrinsically by fear of failure rather than the need to learn and understand.

Students' learning difficulties can often be attributed to ineffective or inappropriate cognitive processes (Herreid, 2003). Earlier, Ramsden (1995) contended that approaches to learning are associated with learning outcomes. According to Novak and Gowin, meaningful learning occurs when individuals choose to relate new knowledge to relevant concepts and propositions they already know (Novak & Gowin, 1984). This calls for commitment on the part of the learner to link new concepts with higher order and more inclusive concepts that are already understood by the learner that can serve to anchor new learning and assimilate new ideas (Novak, 1998).

The persistently low enrollment in mathematics-oriented courses particularly in tertiary institutions have aroused concern of mathematics educators, researchers and policy makers the world over (Changeiywo, 2001; Githua, 2002). As a result most countries are seeking to improve their mathematics education standards by promoting programs that not only enhances effective acquisition of rapidly growing bodies of mathematics knowledge in a well organized framework, but also promotes the learners' capability to learn mathematics meaningfully (Novak, 1998). In practice, while the preponderance of scientific effort swirls around experimental achievements, conceptual achievements continue to be astoundingly important in the overall advancement of mathematics (Wagner & Benavente-McEnery, 2006). If mathematics education aims at preparing students who can think logically and conceptually; solve traditional as well as novel mathematics problems; work efficiently with confidence and accuracy; use meaningful problem solving strategies and are committed to pursuing the study of mathematics; then the focus should be on teaching for understanding rather than students memorising mathematics facts, skills, concepts, principles and strategies (Cooper & Robinson, 2000).

Many students in Kenya perform poorly in mathematics. The poor performance is mainly attributed to poor teaching and learning strategies. It is this poor performance that prompted the Government of Kenya through the Ministry of Education, with assistance of the Government of Japan through Japan International Co-operation Agency (JICA), to initiate a program on Strengthening of Mathematics and Science in Secondary School Education (SMASSE, 2003).

The importance of good teaching cannot be overemphasized. Good teaching encourages high quality learning (Ramsden, 1995). According to Mondoh (2000), students' difficulties in solving problems in mathematics may be traced to: poor understanding of the basic concepts, dependence on algorithms, and inability to apply what they knew, among others. The teaching of mathematics is not just about dispensing rules, definitions and algorithms for students to memorize. There is need to engage students as active participants through discussions and collaboration in problem solving among themselves. If students are given the opportunity to explain or clarify mathematical ideas, more meaningful learning results. Lau (2009) alludes that the mathematics skills required for the youth of today and the adults of tomorrow to function in the workplace are distinct from that for the youth and adults of yesterday. In terms of the 21st century pedagogy, the development of education now requires teaching strategies that emphasize students' involvement (Silva, 2009). Much success lies in students being able to communicate, share and use information to solve mathematical problems. According to Johnson and Johnson (1995), to achieve success in learning mathematics, learners should be given the opportunity to communicate mathematically, reason mathematically, and develop self confidence to solve mathematics problems.

Successful mathematics teaching is associated with explicit teaching of a coherent conceptual framework rather than simply involving students in activities and hoping that meaningful learning results. Thus it is important that mathematics teaching focuses on the quality of understanding rather than on the quantity of information presented. Unfocussed or purposeless activity in the classroom leads to little if any learning. Duffy and Jonassen (1992) argue that teachers should develop instructional strategies that engage learners actively in the process of knowledge construction to enable them learn meaningfully. Learning is considered to be an active, constructive, cumulative, self-regulated and goal-oriented process in which the learner plays a critical role (Trowbridge & Bybee, 2004). There is need to develop teaching strategies that conform to this new perception of learning to enhance meaningful learning.

An analysis of the KCSE examination question papers indicates that questions on Commercial Arithmetics keep recurring year after year, yet no marked improvement has been realised in terms of student performance in the topic even as the general performance in mathematics remains poor (KNEC, 2010). This suggests that students have a problem with this topic. The poor performance depicted by students in this topic portrays inadequate understanding of concepts in it. Teachers have been blamed for using inappropriate instructional techniques in teaching this topic. Techniques that promote student-centred learning are seldom used. This is due to poor instructional approaches used in teaching mathematics (Mondoh & Yadav, 1998; Githua, 2001; Changeiywo, 2001; KNEC, 2010). It is however important that students perform well in this topic since Commercial Arithmetics gives useful information applied in daily life at home, in accounts and in commerce (KICD, 2001).

In Kenya, previous studies on performance in mathematics education concentrated on the direct effects of students' background factors and school environment, students' attitudes and type of instruction (Kirembu, 1991; Makau & Coombe, 1994). Mondoh (1995) identified teaching effectiveness, which is influenced by the teaching approach, as the most significant variable in mathematics achievement.

Problem solving instructions in schools do not emphasise techniques used by skilful problem solvers. In secondary school mathematics courses, the emphasis is usually on problem-specific procedures and mathematical manipulations to help students get answers, rather than the application of powerful ideals and generalisable procedures that could be applied across a wide range of contexts. The lack of emphasis on qualitative reasoning and integrating conceptual knowledge within problem solving instruction encourages rote memorisation of procedures and formulaic approaches that do little to

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foster conceptual understanding. The problems that students solve illustrate a single path to a single answer; the notion that a problem may have multiple solutions or multiple paths to a solution is not stressed. Anderson and Roth (1989) observed that at the primary school level, the solutions to the “problems” that pupils are given usually require only single answer. In respect to this, pupils do not solve problems but rather answer questions. Moreover, students are not given open-ended problems that require to be broken into smaller sub-problems, to device mathematical methods for answering the sub-problems, or to summarise the knowledge learned from solving problems in a form that makes it conducive to apply it in novel contexts.

Problem Solving Approach (PSA) has been widely accepted as the way to teach vocational agriculture. On effects of level of PSA to teaching on students’ achievement and retention, Boone (1990) found that students’ level of achievement and retention was highest when PSA to teach was used. In the same study, Boone found that for high level cognitive items, students taught by PSA exhibited lower achievement loss than those taught by subject matter approach. In an earlier study, Boone (1988) found that high school agriculture students taught using PSA first in an instructional series had higher achievement scores than those taught first using a subject matter approach. Consequently to achieve effective learning and good performance in mathematics, the topic of Commercial Arithmetics need to be taught using student-centred approach. Zechariah (2010) contends that instructional methods employed by the teacher play a significant role in the acquisition of skills and meaningful learning. Instructional methods such as lecture make students become passive and have less interaction with each other in doing tasks. Changeiywo (2001) asserts that the lecture method adopted in schools makes students to be isolated from one another, leading to a high failure rate in sciences and mathematics. Changeiywo is of the view that positive changes take place when a teacher changes the teaching method toward a more student-centred approach. Consequently, an alternative method for the delivery of mathematics knowledge is PSA.

According to Mangle (2008), PSA involves students working in small groups to achieve a common goal, under conditions of positive interdependence, individual accountability, appropriate use of collaborative skills and face-to-face interactions. PSA is the instructional use of small groups through which students work together to maximize their own and each others’ learning. Problem solving has its foundation in social-constructivist perspectives of learning. In this approach, the classroom environment is characterized by co-operative tasks and incentives structures and by small group activities. It can be used to teach ‘hard’ topics in mathematics and also help teachers to accomplish important social learning and human relations goals. Mangle provides benefits on the use of the PSA on students’ achievement in mathematics as: students achieve higher grades; develop positive attitude towards mathematics and their social skills are enhanced. PSA also promotes deep learning of materials and help students to achieve better results in mathematics.

PSA has been shown to lead to improved achievement in mathematics to senior students and those in colleges. Samuelsson (2008) found that PSA teaching approach is more effective than the conventional methods in the academic success of students. Segzin (2009) reported that in PSA sessions, students tend to enjoy mathematics, and this enjoyment motivates them to learn. Several researches on PSA have been on senior students and those in colleges in the Western environment. Hence, it was less clear whether PSA could be successfully applied to secondary school students in other countries in which social, religious, educational, and cultural practices are different from those of the Western countries. It is against this background that the current study investigated the influence of PSA on students’ mathematics achievement in Commercial Arithmetics in Kenya.

From the foregoing, none of the studies so far sought to find out how PSA influences students’ mathematics achievement with an aim of promoting meaningful learning. In an attempt to fill this gap, the current study investigated the influence of PSA on secondary school students’ mathematics achievement in Commercial Arithmetics in secondary schools in Vihiga County.

Purpose of the Study:

The purpose of this study was to investigate the influence of Problem Solving Approach (PSA) on secondary school students’ mathematics achievement in Commercial Arithmetics compared to traditional mathematics teaching approach.

Objective of the Study:

The objective of the study was to determine the influence Problem Solving Approach (PSA) has on students’ achievement in Commercial Arithmetics as compared to conventional methods.

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Hypothesis of the Study:

The following null hypothesis was tested at an alpha level of 0.05:

HO_1: There is no significant difference between the achievement scores of students who are taught using PSA and those taught by conventional methods.

Significance of the Study:

The findings of the present study would contribute to both theory and practice of mathematics education in Kenya's secondary schools. In terms of theoretical value, the findings would help teacher educators at the teacher training colleges and universities and curriculum developers at the Kenya Institute of Curriculum Development (KICD) to understand the dynamic components of PSA: face-to-face positive interaction, positive interdependence, individual accountability, and collaborative skills that make it an effective instructional method that should be recommended for teaching students in secondary schools. This will assist in the review of the secondary school mathematics' syllabi and instructional objectives.

In terms of practical value, the findings would help heads of mathematics departments and mathematics teachers and national, county and sub-county quality assurance officers to emphasize on using PSA during their routine duties to improve on academic achievement in mathematics. The teachers are likely to teach in a stimulating and motivating manner, catering for both girls and boys fairly, thus producing better results among their students. Students stand to gain higher levels of achievement, to build cross-ethnic friendship, to experience and enhance mathematics self-concept development, to build lifelong interaction and communication skills and master the habits of mind (critical, creative and self-regulated) needed in society if they learn in PSA environment.

2. RELATED LITERATURE

The Principles and Standard for School Mathematics (NCTM, 2000) describes Problem Solving Approach as using interesting and well-selected problems to launch mathematical lessons and engage students. In this way, new ideas, techniques and mathematical relationships emerge and become the focus of discussion. Good problems can inspire the exploration of important mathematical ideas, nurture persistence, and reinforce the need to understand and use various strategies, mathematical properties, and relationships. Problem solving is thus the process of confronting a novel situation, formulating connections between the given facts, identifying the goal, and exploring possible strategies for reaching the goal. The aim of this approach is acquisition of information that is based on facts (Yuzhi,2003; Mangle, 2008).

PSA is a constructivist teaching model based on the assumption that learning is a product of cognitive and social interactions originating in a problem focused environment (Greeno *et al*, 1996). The theoretical philosophy of this approach is derived from John Dewey and discovery learning (Rhem, 1998). Fundamentally, PSA is an educational method in which students develop critical thinking and problem-solving skills in addition to developing an understanding of grasping essential concepts through the analysis of real-life problems (Duch, 1995). Learning takes place throughout a process where learners solve problems in groups. Barrows (1996) labels the main characteristics of PSA as: learning is student-centred and takes shape in small groups of students; teacher act as moderator and facilitator; the problems provide motivation for learning and organizational focus as well as the basis for the advance in problem-solving skills; and self-directed learning aids the acquisition of new information. Besides equipping students with knowledge, PSA could also be employed to improve their problem solving skills, critical and creative thinking abilities, lifelong learning aptitudes, communication skills, group cooperation, adaptation to change and self-evaluation abilities, and enables them to build a far more positive approach to learning (Albanese and Mitchell, 1993).

In PSA, students act as professionals (Gallagher *et al.,* 1999). They are confronted with problems that require clear defining and well structuring, developing hypothesis, assessing, analysis, utilizing data from different sources, revising initial hypothesis as the data collected, developing and justifying solutions based on evidence and reasoning. PSA has been used as an educational tool to enhance learning as a relevant and practical experience, to have students' problem solving skills and to promote students' learning skills. Eng (2001) opined PSA as a philosophy aims to design and deliver a total learning that is holistic to student-centred and student empowerment. Presenting the students with a problem, gives them opportunity to take risks, to adopt new understandings, to apply knowledge to work in context and to enjoy the thrill of being discovers.

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Tick (2007) notes that in the student-centred learning environment that is desirable for PSA, the student is the central figure of the learning-teaching process. The learning objective is not the reproduction, recall and learning of passively received learning material. Rather, it is the active and creative engagement of students in group work and in individual study, thus transferring the skills and knowledge. The individual, autonomous self-directed learning gives the freedom to the learner to decide individually and consciously on the learning strategy and on the time scale to follow. Students have the opportunity to express their ideas and justify their answers verbally. They also have opportunities to engage in cognitively demanding questions (Hiebert & Wearne, 1993).

In PSA, the teacher acts as a facilitator. Roh (2003) argues that within problem solving learning environments, teachers' instructional abilities are more critical than in the traditional teacher-centred classrooms. Beyond presenting knowledge to the students, teachers must engage students in marshalling information and using their knowledge in applied and real settings. In teaching through problem solving, the discussion of a problem and its alternative solution takes longer than the demonstration of a routine classroom activity. Hiebert and Wearne (1993) found that classrooms with a primary focus on teaching through problem solving used fewer problems and spend more time on each of them compared to those classrooms without a primary focus on problem solving. Moreover, in problem solving classrooms, teachers ask more conceptually-oriented questions and fewer recall questions than teachers in the conventional classrooms. They also decide the aspects of a task to highlight, how to organize and orchestrate the work of students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge (Stigler & Hierbert, 1999). Thus it is the teacher's role to develop students' reasoning skills. As Weber (2008) avers, "To lead students to develop accurate criteria for what constitutes a good argument, the teacher must have a solid understanding of these criteria" (p. 432).

Learning takes place during the process of problem solving. As students solve problems, they can use any approach they can think of, draw on any piece of knowledge they have learned, and justify their ideas in ways they feel are convincing. The learning environment provides a natural setting for students to present various solutions to their group or class and learn mathematics through social interactions, meaningful negotiations, and reaching shared understanding. Such activities help students clarify their ideas and acquire different perspectives of the concept or idea they are learning (Lester & Charles, 2003).

PSA has important cognitive learning outcomes such as subject achievement, retention, problem-solving skills, learning strategies, approaches to learning (Berkel and Dolmans, 2006; Chin and Chia, 2004). Problem-based tutorial groups positively influence learning. In studies focusing on the cognitive effects of small groups PSA, activation of prior knowledge, recall of information, causal reasoning or theory building, cognitive conflicts leading to conceptual change and collaborative learning construction take place during discussions (Dolmans and Schmidt, 2006). In PSA, students follow a certain pattern of exploration which begins with the consideration of a problem consisting of occurrences that need explanations. During discussion with peers in groups, students try to identify the fundamental principles or processes. Students then stimulate their existing knowledge and find that they need to undertake further study in certain areas. As a result of this, students research the necessary points and then discuss their findings and difficulties within their groups. The discussions held in groups contribute to students' cognitive learning positively (Dolmans *et al.*, 2001).

PSA impacts students' motivation for learning optimistically. A certain cognitive process (i.e. intrinsic interest in subject matter) is facilitated by the process entailed in PSA (Schmidt, 1993). By discussing the subject matter in groups, students become engaged which in turn influences their inherent interest in the subject matter (Dolmans & Schmidt, 2006). Students' intrinsic interest motivates them to develop a full understanding of all the components needed for its solution (Grooves, 2005). Consequently, these cognitive and motivational benefits of PSA have a positive resultant impact on student's academic achievement.

According to Dart *et al.* (2000), PSA produces deep learning which is a modernist method where the leaner actively participates in the learning task so as to reshape the knowledge provided. The surface learning is a product of the conventional method where the learner is completely passive waiting for the teacher to transfer the information directly. Researches have proved that students get influenced by their perceptions of the learning environment when selecting an approach to learning (Trigwell *et al*, 2000; Wasike, 2003; Mayya *et al.*, 2004). In earlier studies, Raimsden and Entwistle (2010) reported that teaching characteristics such as the methods of learning employed in classes, the teacher's enthusiasm, the level of the knowledge being taught and the pace of progression have a great impact on students' achievement. Margetison (2008) noted that conventional methods of teaching encourage the leaner to adopt the surface learning approach; and that it is PSA method that integrates the four vital elements of the deep learning approach; that is a well-structured knowledge database, active learning, interaction through co-operation and the conditions planned in a way to increase intrinsic motivation.

Mathematical problems are well structured in that they are clearly stated, have known solutions and are evaluated against well known undisputed criteria. Biehler and Snowman (1997) indicate that mathematical problems have given information, obstacles and a goal. According to Polya (1973) the four steps that can help a learner to successfully solve mathematical problems are: identification of the problem, which depends on curiosity and interest of the learner in the subject matter; understanding of the nature of the problem based on specific-domain knowledge and familiarity with problem types; recall of mathematical facts and consultation with other relevant source for the required information in a problem; and formulation and implementation of solution to a problem through; use of algorithms, heuristics, study of worked examples, solution of similar but simpler examples, solving analogous problems, and evaluating the solution by estimating or checking its solution.

In support to Polya's four-step sequence, Kelly, Lang and Pagliaro (2003) identified eight problem solving strategies namely: identifying the target goal (what is to be solved), making a plan, identifying the key information, evaluating one's plan and solution, generating and testing hypothesis, estimation, trial-and-error and dividing a problem into sub-problems (two or more procedural operations). Problem-solving strategies are key to enabling students to continue their learning far beyond the classroom, leading them to become autonomous learners. Successful problem-solvers draw on a variety of strategies, use knowledge of patterns within their topic, make certain that they understand a problem before attempting a solution, and use effective self-regulation during the process. Once they understand the problem, they develop an initial plan, carry it out and reflect on the success of the solution (Polya, 1973; Kelly *et al.*, 2003).

In regard to mathematics specifically, a difficulty in some curricular is that algorithms are taught out of context. Lochhead and Zietsman (2001) argue that teaching must be done within the context in order to avoid students' perfunctory performance on algorithms alone. They further assert that much emphasis is on general-purpose strategies that can be applied across a range of mathematical contexts. Beyer (1988, 2001) supports Polya's four-step sequence of introducing mathematical problem solving. The teachers reinforce this strategy and elaborate upon it as student progress through the classes, using it as a framework for a variety of solution plans and formulae.

Cook (2001) stresses tasks that engage students in problem-solving and mathematics reasoning. Cook argues that quality rather than quantity should rule the day in problems that are thought-provoking and those that challenge students' curiosity. Students can also gain from learning strategies such as: trial-and-error, drawing a diagram or model, process of elimination, looking for patterns, simplifying the problem, working backwards, organizing information and then writing an equation. Lochhead and Zietsman (2001) contend that good problem-solvers have these strategies as part of their repertoire. Besides, they have a positive and determined attitude about problem-solving, and awareness in the sense of understanding *how* they solved the problems. This study used Polya's problem solving heuristics during mathematics instruction.

Studies involving elementary students showed that students taught through the PSA had higher levels of mathematical understanding and problem solving skills on a computation test than those taught with the conventional methods (Carpenter, 1998; Fuson *et al.*, 2000). Other studies involving middle school students (Ridgeway *et al.*, 2002; Romberg & Shafer, 2002) revealed that students taught with the problem based instruction had higher levels of mathematical understanding than the students taught by the traditional instruction. Earlier, Wood and Sellers (1997) found that students who received problem-centred mathematics-instruction had significantly higher achievement on standard achievement measures and better conceptual understanding than did those students who had received the traditional instruction. In studies involving pre-service Physics teachers, those taught through problem based learning (herein referred to as the PSA) instruction had higher levels of achievement in comparison to those who received instruction through the traditional methods (Segzin, 2009).

Although the literature reviewed supports the benefits of PSA, none of the studies focused on the influence of the PSA on students' achievement in Vihiga County schools. Thus, this study investigated on the influence of PSA on students' achievement in mathematics in Vihiga County schools.

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3. RESEARCH METHODOLOGY

Research Design:

The present study adopted Solomon's Four Group Design that employed the quasi-experimental procedures. This is because secondary schools classes once constituted exist as intact groups and school authorities do not allow such classes to be broken up and re-constituted for research purposes (Gall, Borg & Gall, 1996). Thus it was not possible to assign individual students randomly to groups as required in true experimental designs. The schools selected were however randomly assigned to the treatment and control conditions as intact groups. The pre-test – post-test approach was used to partially eliminate the initial differences between the experimental and control groups (Gibbon & Herman, 1997). The design is considered rigorous enough for experimental and quasi-experimental studies. This is because it provides effective and efficient tools for determining cause and effect relationship. It also provides adequate control of other variables that may contaminate the validity of the study. The design helped to achieve four main intentions, namely: to assess the effect of the experimental treatment relative to the control condition; to assess the interaction between pre-test and treatment condition; to assess the effect of the pre-test relative to no pre-test and to assess the homogeneity of the groups before administration of the treatment (Borg & Gall, 1989).

According to Sharma (2002), the Solomon's Four Group Design is a particular strong quasi-experimental procedure. However, it is important that there is opportunity for both a pre-test and post-test in both the treatment and the control groups. The Solomon Four Group Design that employs the Quasi-experimental research design procedures controls for all major threats to internal validity except those associated with interaction of: selection and history; selection and maturation; and selection and instrumentation (Gibbon & Herman, 1997).

In this study, no major event was observed in any of the sample schools that would have introduced interaction between selection and history. However, to control for interaction between selection and maturation, the schools were randomly assigned to the control and treatment groups. The conditions under which the instruments were administered were also kept as similar as possible across the schools to control for interaction between selection and instrumentation (Sharma, 2002). An instructional manual for teachers was developed based on Kenya Institute of Curriculum Development (KICD) approved mathematics syllabus (2002). The manual was used by teachers teaching the experimental groups to ensure that there was uniformity in exposure of students to intervention. Furthermore, all the teachers involved in the study adopted the same schemes of work and similar sequence in covering the content on Commercial Arithmetics in all the schools involved in the study. Hence, there was reasonable control of the threats to internal validity of the study. The design is shown in Table 1

Table 1: Solomon's Four Group Design

Groups	Pre-test	Treatment	Post-test
1	O_1	X (Problem Solving Approach)	O_2
2	O_3	C (Conventional Methods)	O_4
3		X (Problem Solving Approach)	O_5
4		C (Conventional methods)	O_6

Source: Adapted from Gibbon and Herman (1997)

In this design, subjects were assigned randomly to four groups. Groups 1 and 3 received the experimental treatment (X) that was the use of the Problem Solving Approach (PSA) in teaching. Group 1 received a pre-test (O_1) and group 2 received a pre-test (O_3). Groups 2 and 4 constituted the control and use of conventional methods in teaching. Finally all the four groups received post-test (O_2, O_4, O_5 & O_6). The research design is a combination of two group designs, the post-test only and the pre-test - post-test which control extraneous variables of testing, history and maturation (Gibbon & Herman, 1997). The subsequent section describes the treatment.

Description of Treatment:

The conventionally-designed mathematics instruction was based upon lessons employing lecture/questioning method to teach the topic of Commercial Arithmetics. The teaching strategy depended upon teacher explanations, discussions and textbooks. The teacher treated the entire class as a unit, wrote notes on the blackboard about the definition of different terminology and solved most of the problems in the topic. After the teacher explanation, the concepts were discussed, recapitulated by the teacher's questions. The direction of communication in the classroom was from the teacher to the student. The teacher was the focal point of the discussion and dispenser of the mathematical knowledge.

In the experimental groups, before the treatment, small groups consisting of five to eight students were formed. Then, the students and the teachers were trained to use the PSA. During the treatment, the students worked in small groups and dealt with ill-structured problems. Every member of the group had some responsibilities. Students participated actively in the group discussions. They had to share their knowledge, express their ideas and experiences with each other while searching a solution to the problem. Each of them had to be sensitive to the needs and feelings of the other group members. Apart from the group work, each student had to conduct an independent study and be able to represent, communicate and evaluate his/her learning at both individual and group levels.

During the PSA sessions, the teacher organised the groups and created a purposeful and co-operative atmosphere. The teacher ensured that students had control of the discussion. When guidance was needed, the teacher asked open-ended, very general questions and gave ample opportunity to students to the focus on the goal. The teacher encouraged critical thinking. At the end of the PSA implementation, the students evaluated each other with respect to participation, preparation, interpersonal skills and contribution to group progress. In this way, it was expected that students would become aware of the role expected from them both individually and as a group. The experiment lasted for three weeks. After the three weeks of treatment, post-tests were administered.

Target Population:

The target population of the current study consisted of all Form three mathematics students from public schools in Vihiga County. The county was chosen for this study because there was no study on the influence of the teaching strategy on students' achievement in terms of conceptual understanding in Commercial Arithmetics. This has been blamed on the teaching strategies and to some extend on low mathematics self-concept held by secondary school students in the county; a claim that lacked empirical evidence to support it. Nonetheless, a good teaching strategy encourages high quality students learning (Ramsden, 1995). Thus there was need to explore for innovative teaching strategies that will help promote cognitive characteristics of the learners if the low mathematics achievement has to be reversed.

Form Three students were chosen because the topic Commercial Arithmetics selected for the study is taught at this level (Kenya Institute of Curriculum Development [KICD], 2002), they could express their mathematical ideas in written form (Githua, 2002). The county has 114 schools: 2 national schools, 10 county schools, 97 district schools and 5 private schools. National, county and district schools were selected. This is because students' achievement in mathematics is poor in the county (Education Office Vihiga, 2010). There were 109 such schools with a population of 10,555 students.

Sampling Procedure and Sample Size:

The sampling frame consisted of all national, county and district secondary schools in Vihiga County. The first stage was the purposive selection of Vihiga County and the type of school (i.e. national, county and district schools) included in the study sample. Purposive sampling was used to select the two national schools that participated in the study. The remaining schools were stratified into boys' only, girls' only and co-educational schools. Ten schools were then drawn out of the remaining 107 schools. Because of the smaller number of schools to sample from, balloting method was employed. This involved assigning a numeral to each of the 107 schools, placing the numbers in a container and then picking a number at random without replacement. Schools corresponding to the numbers picked and having at least three streams at the Form three level were included in the study sample.

According to Mugenda and Mugenda (2003), at least 30 students per group are required for experimental research. They are of the view that the sample size should be sufficiently large enough to allow accurate interpretation of the results as well as ensuring that the data is manageable. Twelve schools were sampled. One class (each with at least three streams) from each school was included in the study sample. The twelve classes in the twelve schools were assigned to the four groups in the Solomon four-group experimental design. Although it was assumed that the average enrolment was forty students per stream, giving the approximate sample size of the study as 1440 students, the actual sample size that participated was 1663 students. During data coding, it was found that some students had either incomplete data and/or missed some test. This reduced the sample size for data analysis to 1459 students. These subjects were used in their twelve intact classes in the twelve schools that were assigned to experimental groups 1 and 3, with 367 and 360 students respectively; and control groups 2 and 4, with 344 and 388 students respectively.

Research Instruments:

According to Sharma (2002), no single method of data acquisition is sufficient. Thus, using more than one method of collecting data is recommended. As such, two instruments namely; Mathematics Achievement Test 1 (MAT 1) and Mathematics Achievement Test 2 (MAT 2) were used to collect data to meet the objective of the study. They were developed and pilot tested prior to the actual conduct of the study. The items in the tests were based on the table of specification designed for the topic Commercial Arithmetics against the three cognitive levels (knowledge, application & comprehension) as stated in the Blooms taxonomy. MAT 1 was used as a pre-test and had items on the topic Commercial Arithmetics covered at the Form one level. The topic is covered in the Form one syllabus and it builds the foundation for Commercial Arithmetics topic taught at the Form Three level which was the focus of this study. It was administered during the first week of the study. Its purpose was to establish the entry behaviour of the learners before the treatment. MAT 2 was given to six University lecturers who gave their comments after studying the items. Language and other noticeable problems were corrected. The test was then given to three secondary school mathematics teachers who were also examiners with the KNEC. They reviewed the items and made their comments. Their feedback was used to improve the test items. The improved test (MAT 2) contained fifteen items and was used as a post-test. It was used to assess Form three students' achievement in Commercial Arithmetics after the treatment. It was administered after the treatment when all the lessons had been taught. The instruments were pilot tested on 42 Form Three students in Vihiga County that did not participate in the study. The students had similar socio-backgrounds as those that were used in the final study. The pilot study aimed at assessing the appropriateness of the instruments.

Validity of Instruments:

MAT 1 and MAT 2 were assessed for content and face validity. This was done by two experienced secondary school mathematics teachers, the two academic supervisors and two mathematics educators from the Department of Science and Mathematics Education at Masinde Muliro University of Science and Technology. Each panel member assessed the items in MAT 1 and MAT 2 for content coverage and level of difficult. Their responses were measured on a five-point Likert scale (see Appendix II). They were scored and transcribed into a percentage score. An average score of above 70% for face and content validity implied that the instrument was appropriate. The averages of the responses of the face and content validity of each of the instruments are as shown in Table 2.

Table 2: Summary of Assessment of Instruments' Validity by Percentage

Instruments	Type of Validity	Mathematics Teachers	Academic Supervisors	Mathematics Educators	Average Percentage	Conclusion
MAT 1	Face	86	74	78	79.33	Appropriate
	Content	88	85	94	89.00	Appropriate
MAT 2	Face	87	82	86	85.00	Appropriate
	Content	90	88	92	90.00	Appropriate

Reliability of Instruments:

The reliabilities of MAT 1 and MAT 2 were ascertained using test-retest method. The instruments were administered twice to the same group of students. There was a two-week time lapse between the first administration and the second one. The correlation coefficients were ascertained using Cronbach's Coefficient Alpha method (Gall, Borg & Gall, 1996). Correlation coefficients of 0.795 and 0.872 were obtained for MAT 1 and MAT 2 respectively. These values of correlation coefficients were acceptable for the study, since Fraenkel and Warren (1990) assert that an alpha value above 0.70 is considered appropriate to make possible group predictions that are sufficiently accurate.

Development and Use of Instructional Materials:

The instructional materials used in the study were based on the KICD approved mathematics syllabus (KICD, 2002). The Secondary Mathematics Students' Book Three by KICD was used as the textbook in the control group. In the experimental group, the PSA module was used. The module had two manuals, one for the teacher (Teachers' manual) and one for the students (Students' manual). The Teachers' manual was derived from the Students' manual. These manuals contained the mathematics content that was covered during the instructional process. The Teachers' manual is a written copy of all of the steps a student needs to take during the lesson (that is, defining the problem, determining the learning goals, reaching new information by researching, doing numerical analysis of the problem etcetera). In the Students' manual, the previously mentioned parts were left empty for the students to complete.

In the beginning of the PSA sessions, the copies of the modules were distributed to each student and teacher. The teachers' instructional manual generally described the methodology that was used in teaching the topic of Commercial Arithmetics. These included a detail description of the specific teaching approach tested in the present study; the instructional method and teaching/learning activities as well as how the specific mathematics concepts and skills were to be presented in the topic. Detail description of all the procedures were necessary to ensure uniformity as much as possible among the teachers involved in the study.

Data Collection Procedures:

Before the treatment started, the research assistants from participating schools were inducted for a period of two days by the researcher as pertains to the use of the PSA and conventional methods. This period was appropriate because the teachers involved in teaching the experimental and control groups were trained. They trained the students in the experimental groups pertaining to the requirements and use of PSA for a period of three days. To minimize differences in teachers' teaching approaches and ensure that emphasis was given to certain aspects of teaching, the researcher met with all the teachers involved in the study on weekly basis. In the meeting discussions on the content, problems as well as instructional approaches applied was done. The researcher wanted to make sure that the quality of teaching was decent and acceptable. Teachers in the experimental groups were also issued with instructional manuals specifically developed for the topic of Commercial Arithmetics.

After the induction period, the research assistants administered a ninety-minute MAT 1 to students in groups 1 and 2. The MAT 1 scripts were collected and scored for three days in each respective school by the researcher and his assistants. The pre-test scores were used to assess the entry level and homogeneity of the students in the randomly assigned experimental and control groups. The researcher and his assistants taught groups 1 and 3 the topic Commercial Arithmetics using PSA for a treatment period of three weeks. Groups 2 and 4 were taught the same topic using conventional methods where learning was mainly teacher-centred. It entailed the use of lectures, question/answer techniques, teacher-led discussions and worked-out class examples that were mainly teacher-dominated.

Two days after the treatment period, the researcher and his assistants administered a ninety-minute MAT 2 to all the four groups at the same time. The researcher visited the schools after two days to collect the data that was taken to a central marking point. The rate of return of data collection was 87.73 percent. The researcher with the help of the research assistants thus scored and coded the collected data. To ensure uniformity in the marking, the MAT 2 scripts were scored using the belting system as currently advocated by the KNEC. The pre-test and post-test results scores were then correlated and analysed.

Data Analysis Techniques:

The data obtained in the study constituted of MAT 1 pre-test scores and MAT 2 post-test scores of the experimental and control groups. The descriptive statistical tests that were done comprised of percentages, means and standard deviations. The inferential statistical tests; the t-test and the Analysis of Variance (ANOVA) were used to analyse data at an alpha level (α) of 0.05. The t-test was used to analyse the pre-test and the post-test influence. It was also used to compare whether students' mean scores were significantly different, based on the pre-test scores of experimental group 1 and control group 2. A comparison of mean scores and tests for significance difference between experimental and control group scores was done using ANOVA. An F-test was used to determine whether the differences were significant.

4. RESULTS

Results of Pre-tests:

The Solomon Four-Group Design used in this study enabled the researcher to have two groups sit for pre-tests. The aim for pre-testing was to ascertain whether or not the students selected to participate in this study had comparable characteristics before presenting the topic Commercial Arithmetics. To achieve this aim, the students in groups 1 and 2 sat

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for the pre-test MAT 1. This made it possible for the researcher to: assess whether there was any interaction between the pre-test and the treatment conditions; assess the influence of the pre-test relative to no pre-test; and assess the similarity of the groups before the administration of the treatment (Borg & Gall, 1989).

A total of 711 students were administered with pre-test MAT 1, of which 367 were in group 1 and 344 in group 2. Table 3 shows the t-test of the pre-test scores on the MAT 1.

Table 3: Independent Samples t-test of the Pre-test Scores on MAT 1

Experimental Group 1, N = 367 Control Group 2, N = 344

Variable	Group	Mean	SD	t-value	P-value
MAT 1	1	37.66[a]	8.18	0.313*	0.754
	2	37.88[a]	10.37		

Notes:

[a,] denotes similar mean scores * Not significant at $p<0.05$ level SD: Standard Deviation

MAT 1 Maximum Mean Score = 100 df = (1,709)

From Table 3, the experimental group 1 scored a mean of 37.66 and the control group had a mean of 37.88 in MAT 1. From the results, the pre-test mean scores of both groups (1 & 2) obtained were similar on MAT 1. The t-test results analysis reveal that the pre-test mean scores for groups 1 and 2 on MAT 1 measure are not statistically different since the t-value for MAT 1 (0.313), is not significant at 0.05 α-level, df = (1, 709).

An examination of the results in Table 3 indicate that the pre-test mean scores for experimental group 1 and control group 2 on MAT 1 are not statistically different at 0.05 α-level. From the results presented in Tables 3, it suffices that the pre-test MAT 1 mean scores of students in the experimental group 1 and the control group 2 are not statistically different at 0.05 α-level. Also, it was assumed that the students were randomly assigned to classes at the Form one level and they continued in their intact classes until Form four. These indicate that the four groups used in the study were comparable and had similar entry behaviour, hence homogeneous. This made them suitable for the study.

Influence of PSA on Students' Achievement in Commercial Arithmetics:

In order to determine the influence of PSA on students' achievement in Commercial Arithmetics, an analysis of the students' post-test MAT 2 scores was carried out. Hypothesis one (HO_1) of the study sought to find out whether there was any significant difference between the achievement of students who were taught using PSA and those who were taught by conventional methods. Table 4 shows the MAT 2 post-test mean scores obtained by the students in the four groups.

Table 4: MAT 2 Post-test Mean Scores Obtained by Students in the Four Groups

Group	N	Mean Score	SD
1	367	52.47[a]	10.35
2	344	39.55[b]	11.48
3	360	53.93[a]	8.99
4	388	38.41[b]	12.82
Total	**1459**	**40.05**	**13.14**

Notes: [a, b] denote similar mean scores Maximum Mean Score = 100

Results in Table 4 show that the MAT 2 post-test mean scores for the experimental groups 1 and 3 (52.47 & 53.93) and that of the control groups 2 and 4 (39.55 & 38.41) respectively, are quite similar. However, the MAT 2 post-test mean scores for the experimental groups 1 and 3 are much higher than that of the control groups 2 and 4. This suggests that the experimental groups performed much better than the control groups in the MAT 2. From Table 4 the highest mean score was attained by group 3 (experimental group 2) followed by group 1 (experimental group 1) then group 2 (control group 1) and finally group 4 (control group 2). These means are presented graphically in Figure 1.

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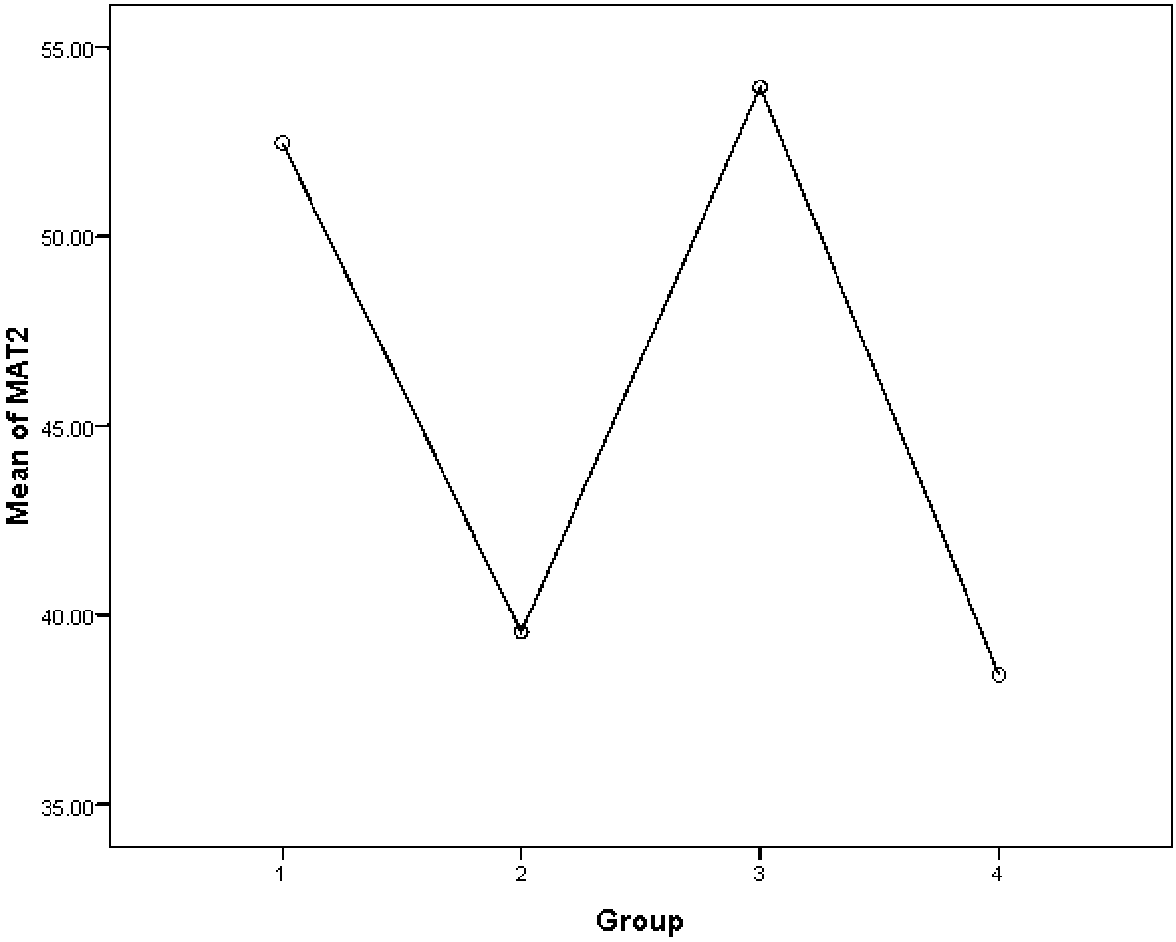

Fig 1: Post-test Means on MAT 2

In order to determine whether the difference in the MAT 2 post-test mean scores was statistically significant, a one-way ANOVA was performed. The results of the one-way ANOVA based on these mean scores are shown in Table 5.

Table 5: ANOVA of the Post-test Scores on the MAT 2

Source of Variation	Sum of Squares	df	Mean Square	F	P-value
Between Groups	74670.61	3	24890.21	204.56*	0.00
Within Groups	177036.22	1455	121.67		
Total	251706.83	1458			

Notes: *Denote significant mean differences at $p< 0.05$ df = (3,1455)

The results of the ANOVA in Table 5 indicate the F-value to be significant at $p<0.05$ since the F-value (204.56) from ANOVA is significant at 0.05 α-level, df = (3, 1455). This implies that the MAT 2 post-test mean scores of the four groups are statistically significant.

Having established that there was a significant difference between the means, it was necessary to carry out further tests on the various combinations of means to find out where the difference occurred (SPSS, 2007). Table 6 shows the results of the Least Significance Difference (LSD) post hoc comparisons. LSD post hoc comparisons were preferred over the others since they could best help establish whether there was a statistically significant difference in achievement of the students who were taught using the PSA and those taught by the conventional methods.

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Table 6: Post Hoc Comparisons of Post-test of MAT 2 Means for the Four Groups

	(I) Group	(J) Group	Mean Difference (I–J)	P-value
LSD	1	2	12.93*	0.000
		3	-1.46	0.075
		4	14.06*	0.000
	2	1	-12.93*	0.000
		3	-14.38*	0.000
		4	1.13	.166
	3	1	1.46	0.075
		2	14.38*	0.000
		4	15.52*	0.000
	4	1	-14.06*	0.000
		2	-1.13	.166
		3	-15.52*	0.000

Note: * = The mean difference is significant at the $p<0.05$ level (2-tailed test)

The results in Table 6 show that the pairs of MAT 2 mean scores of groups 1 and 2, 1 and 4, 2 and 3 and 3 and 4 were significantly different at the 0.05 α-level. However, the mean scores of the experimental groups 1 and 3 and the control groups 2 and 4 were not statistically different. Since the MAT 1 pre-test mean scores indicated that there was no significant differences between the entry levels of the groups involved in the study, then it was not necessary to confirm the post-test results by performing Analysis of Covariance (ANCOVA).

Differences in the MAT 2 post-test mean scores of the experimental groups 1 and 3 and the control groups 2 and 4 were not significant. It is also evident from Table 4, that the MAT 2 post-test mean scores of the control groups 2 and 4 were almost similar and much lower than those of the experimental groups 1 and 3.

The results indicate that the MAT 1 pre-test did not interact significantly with the treatment conditions. If this were the case, the groups, which took the pre-test, would have obtained different results from those that did not take it (Borg & Gall, 1989). The pre-test MAT 1 did not affect the students in the learning of the content. If this were the case, the students who sat for pre-test would have different results from the others. This made the pre-test suitable for the study (Kothari, 1990). The use of PSA resulted in higher students' achievement than the conventional methods since the experimental groups 1 and 3 obtained significantly higher mean scores.

Considering the results presented in Tables 4, 5 and 6, it was found that the post-test mean scores obtained by students in the experimental groups 1 and 3 (52.47 & 53.93 respectively) were not significantly different at p equal to 0.05 α-level. In addition, the mean scores of the control groups 2 and 4 (39.55 & 38.41 respectively) are not significantly different. However, the mean scores obtained by students in the groups 1 and 2, 1 and 4, 2 and 3 and 3 and 4 are significantly different at $p<0.05$. In view of these findings, the null hypothesis HO_1 suggesting that there is no significant difference between the achievement of students who are taught using PSA and those taught by conventional methods is rejected.

A comparison of the students' scores in the pre-test and post-test MAT was carried out. Table 7 shows a summary of these scores together with the mean gain.

Table 7: Comparison of Mean Scores and Mean Gain in MAT 1 and MAT 2

Scale	Overall (N = 711)	Experimental Group 1 (N = 367)	Control Group 2 (N = 344)
Pre-test Mean (MAT 1)	37.77	37.66[a]	37.88[a]
Post-test Mean (MAT 2)	46.22	52.47	39.55
Mean Gain	8.45	14.81	1.67
df	710	366	343
t-value	25.49	39.55	7.40
p-value	0.00	0.00	0.00

Notes: [a] denotes similar mean scores Maximum Mean Score = 100

Results in Table 7 show that the pre-test mean scores for experimental group 1 (37.66) and control group 2 (37.88) are quite similar. After the intervention, the experimental group 1 attained a mean score of 52.47, while the control group 2 that had no intervention got a mean score of 39.55. However, the experimental group 1 had a higher mean gain (14.81) than the control group 2 (1.67). The experimental group thus gained more than the control group. The net difference between the mean gains is 13.14. The overall mean gain on students' achievement in Commercial Arithmetics was 8.45. The paired sample t-test between the pre-test and post-test mean scores reveal that the experimental group 1 gained significantly from the teaching as compared to the control group 2, since the t-value 39.55 for the experimental group is significant at 0.05 α-level, df = (1, 366) while that of the control group 7.40 is significant at 0.05 α-level, df = (1, 343). Hence, the experimental group that was taught using PSA had a higher mean gain than the control group that was taught by the conventional methods. This strengthens the position that the PSA resulted in higher achievement than the conventional methods.

5. DISCUSSION

Results of the Pre-tests:

This study employed the Solomon Four-Group Design. The students were put in four groups such that groups 1 and 3 were the experimental groups while groups 2 and 4 were the control groups. Groups 1 and 2 took the pre-test while groups 3 and 4 did not take the pre-test. Such an arrangement enabled the researcher to determine the presence of any interaction between pre-test and the PSA treatment; determine the influence of the pre-test relative to no pre-test; determine the similarity of the groups before applying the treatment and generalise to the groups which had not received the pre-test (Sharma, 2002).

Sanders and Pinhey (1979) assert that when the two experimental groups (1 & 3) are similar to each other in the post-test as opposed to the two control groups (2 & 4), then the researcher is in a strong position to attribute the differences to the experimental condition. A greater difference in the post-test between the experimental groups 1 and 3 in comparison to that between the control groups 2 and 4 results if the pre-test interacts with the treatment. This is as a result of a sensitisation effect - that means the pre-test facilitates the learning of the experimental group in contrast to the control group. The post-test students' mathematics achievement result in this study did not indicate any interaction between the pre-test and the PSA treatment.

Higher post-test performance by groups 1 and 2 than that of groups 3 and 4 could have been the results if the pre-test provided a practice effect. This is not the case since a comparison of the post-test results of the four groups fails to indicate any practice effect provided by the pre-tests. The results therefore portrayed that the pre-test MAT 1 was suitable for the study.

A comparison of groups 1 and 2 students' pre-test MAT 1 mean scores revealed non-significant differences (Table 3). This results show that the groups were quite similar before the administration of the treatment.

Influence of PSA on Students' Achievement in Commercial Arithmetics:

The researcher found that students who were taught using PSA achieved significantly higher scores in the MAT 2 than those who were taught by conventional methods. This implies that the use of PSA is more effective in enhancing students' achievement than the conventional methods.

The success of PSA as a teaching strategy focused on teaching the mathematics topic Commercial Arithmetics through problem-solving contexts and inquiry-oriented environments, which was characterised by the teacher helping the students to conceptualise mathematical ideas and processes by engaging them in doing mathematics (Begle, 1979). The teacher acted as a facilitator in the teaching-learning process. The teacher structured a conducive learning environment that fostered interaction between students/students and teacher/students, mathematical dialogue and consensus between the students. Consequently, the teacher provided enough information that established the background/intent of the problem; accepted right/wrong answers in a non-evaluative way; guided, coached, asked insightful questions and shared in the process of solving problems. In addition, the teacher knew when it was appropriate to intervene and when to step back and let the students make their own way (Lister & Raymond, 1994). Moreover, the teacher encouraged students to make generalisations about rules and concepts. Further, the success of PSA also constituted problem-solving strategies that engaged learners in problem solving and mathematics reasoning (Kelly *et al.*, 2003). These comprised of: Identifying the target goal; making a plan; identifying the key information; evaluating one's plan of solution; hypothesis generation and testing; estimation; trial-and-error; and dividing a problem into sub-problems. The current study was conducted by incorporating these problem-solving strategies in classroom instruction. Consequently, the higher achievement reported.

The finding of this study is consistent with the findings of PSA instruction research in different subject matters and grade levels. For instance, the research conducted on PSA revealed that science students who received instruction using PSA attained higher achievement scores than those taught by the conventional teaching methods (Chin and Chia, 2004). The success of the PSA model on subject achievement can be attributed to the cognitive and motivational effects. Cognitive effects positively contributing to the ability of students to apply knowledge are stimulated by the PSA. In addition to this, PSA enhances inherent interests (that is, motivational effects) in the subject matter (Dolmans *et al.*, 2001).

The acquisition and structuring of knowledge that takes place in PSA does so through certain cognitive effects. These effects are the initial analysis of the problem and activation of pre-existing knowledge through focused discussion in small groups, embellishment of prior knowledge and active processing of new information, reorganization of knowledge, contextual learning and stirring of curiosity related to the presentation of relevant problems (Schmidt, 1993). Students' active engagement in the PSA process had a positive impact on their learning and this is turn enhanced their achievement in mathematics.

The results obtained in this study show consistency with those of ealier studies claiming that PSA encourages deep learning in students (Coles, 2005; Newble & Clarke, 2006; Shimazoe & Adrich, 2010). These studies also indicate the increased use of meaningful (deep) approaches by the PSA students in relation to the material, while decreased use of reproductive (shallow) approaches. Similar to the outcome of other studies, the PSA students made great progress in their deep approach, while they were on the decline with their surface approach. The finding about the progress in deep approach shows a consistency with the results of studies conducted in different fields. For instance, studies carried out in nursing education (Tiwari *et al.*, 2006) and foreign language education (Mok *et al.*, 2009) proved that PSA led students to adopt a deeper approach to learning.

Margetson (2008) stated that the elements encouraging students to adopt deep approach to learning (that is, well-structured database, active learning, interaction based on cooperation and conditions designed in a way to increase intrinsic motivation) are already embraced in the PSA teaching method. The deep approach describes the active engagement of the student with the instructional material, leading to full exploration of the learning material in order to reach a more profound level of personal understanding. Contrarily, the surface approach shows the use of constant memorization which is conducted in order to remember details primarily for assessment purposes (Entwistle, 2001).

Previous research shows that student's achievement is directly affected by their academic environment (Entwistle & Tait, 2000; Wasike, 2003). The results reveal that as well as factors like overload, students' perception of the suitability of the material, poor teaching; poor rapport with students and lack of self-management opportunity issues lead to the adoption of the surface learning approach (Mayya *et al.*, 2004). In the course of the present study, while only a few students in the conventional classes participated in the teaching-learning process; all the students in the PSA groups were required to review the instructional material, to participate actively in their learning process and interact with their peers and teachers. Throughout the PSA sessions, the students had a bigger tendency towards deep approach learning. As a result, students' active participation in the learning activities carried out in the PSA classes, their use of their own strategies of deep learning and observation of each others' study processes had a positive impact on their achievement.

The results obtained in this study go parallel with the PSA results obtained in studies conducted in various disciplines such as medicine, science and social sciences (Wood, 2003; Kaufman & Mann, 1997; Sahin, 2010). It is clear from literature that PSA leads to the students having a more positive outlook towards the subject matter. This is attributed to the fact that those students' perceptions alter and that their awareness of the relevance of the work increases. Moreover, they are able to compare the task of finding information and developing a solution to solving a mystery (Williams *et al.*, 2000).

Researches in other fields of education attest to the benefits and effectiveness of PSA. In studies involving pre-service physics teachers, those taught using problem based learning (herein referred to as PSA) instruction had higher levels of achievement in comparison to those who received instruction through the traditional methods (Segzin, 2009). Evidence from other researches suggests that PSA as a teaching strategy for promoting meaningful learning; may be particularly useful to students who have traditionally experienced difficulty when learning mathematics (Carpenter *et al.,* 1998; Ridgway *et al.,* 2002; Romberg & Shafer, 2002). In the teaching of agriculture using PSA, Flowers (1986) and Boone (1988) found a significant improvement in the students' achievement, and their results are consistent with the findings of the current study. In addition, the findings of this study concur with the arguments of Montgomery (1990) that conceptualisation in mathematics teaching depends upon practical activity. The results also agree with Hamachek (1995), Okere (1996) and Githua (2002) that subjects that have structured information such as mathematics and sciences, require direct teaching that demand the teacher to review pre-requisites, state lesson objectives, give guided practice accompanied with feedback, allow supervised individual work and finally give review through a question and answer session or problem solving. Further, the current findings support the findings of Mondoh and Yadav (1998) that learners' achievement in mathematics depends on the effective use of modern teaching techniques that emphasises a heuristic approach (in this case – the Problem Solving Approach). Moreover, the findings comes in support of Beyer (2001), Cook (2001), Lochhead and Zietsman (2001) and Kelly *et al.,* (2003), that successful PSA always constitute problem-solving strategies that engage learners in problem-solving and mathematics reasoning.

The current findings of this study are consistent with the findings of Fuson *et al.,* (2000) on the influence of PSA as a teaching strategy on achievement in mathematics in schools in America. Results from Fuson et al., study showed that the mean score on mathematics achievement post-test for the experimental group exceeded that of the control group and the difference was statistically significant. The current findings also parallel results obtained earlier by Wood and Sellers (1997) that students who received problem-centred mathematics-instruction had significantly higher achievement on standard achievement measures and better conceptual understanding than did those students who had received traditional instruction. PSA is significantly better for improving students' performance in conceptual understanding.

The results are also consistent with other studies investigating program that focused on problem solving in small groups (Goods & Gailbraith, 2008; Leiken & Zaslavsky, 2008). Students who worked in the PSA classes were exposed to a higher level of reasoning. In conventional classes, students interact with the teacher, while students working on problem solving activity interact with both their peers and their teachers (Oppendekker & Van Damme, 2006). In addition, active participation and the communication of thought processes with teachers of higher ability were critical factors that enabled the students to develop their conceptual understanding in the subject. The discussions held in groups provided the students with the opportunity to explore variations between their own and their partners' knowledge and thinking, correct mistakes and fill gaps in understanding (Granstrom, 2006). In the problem solving activity, students needed to convince themselves and their partners of the correctness of a particular problem. Thus, PSA that is characterized by communication was positive for students' conceptual understanding.

In teaching using problem solving, learning took place during the process of problem solving. As students solved the problems, they could use any approach they could think of, drew on any piece of knowledge they had learned, and justified their ideas in ways they felt were convincing. The learning environment of teaching using problem solving provided a natural setting for students. This enabled them to present various solutions to their group or class and in the process learned mathematics through social interactions, meaning negotiations, and thus reached shared understanding. Such activities helped students clarify their ideas and acquire different perspectives of the concept or idea they were learning (Lester & Charles, 2003). The act of solving problems helped learners to recognize new relationships among the mathematics concepts and refined their understanding of the existing relationships. Problem solving helped students build explicit links and relations between the mathematics concepts (Lochhead & Zietsman, 2003) and thus it was used as a teaching strategy for groups as well as personal learning. PSA as a teaching and learning strategy as employed in the present study stimulated construction of integrated knowledge structures. The stimulation led to better conceptual understanding and thus promoted meaningful learning among the students.

Working on mathematics activities in small groups in problem solving context makes learning an active process rather than a passive one. Thus there is need for mathematics teachers in Kenya to promote the use of PSA as a teaching/learning strategy that promotes active and meaningful learning in class. This strategy is expected to help students make both cognitive and affective improvements in mathematics learning. PSA as a teaching strategy can facilitate student improvement by providing them with opportunities for effective learning processes such as; active processing of information, self-regulation of the learning, immediate correction and feedback, and social interaction which is one of the key concepts in Vygotsky's theory. These results are in agreement with Novak's (1984, 1998) description of meaningful learning as the establishment of non-arbitrary relations among mathematical concepts in the learners' minds. Moreover, it highlights the importance of mathematics instruction that emphasises identifying key concepts and stresses on teaching mathematics concepts and their relationships (Novak, 1984). Thus, it can be concluded that problem solving involved students who mastered mathematics concepts and skills in actively relating new information to prior knowledge resulting in meaningful learning and consequently higher achievement.

The PSA used in this study has demonstrated a great potential of promoting certain cognitive and affective skills of Form Three students in mathematics. Based on the findings of this study, the issue of students having problems in conceptualising the topic Commercial Arithmetics may be resolved by the use of the PSA. In view of this, the low achievement of students in mathematics in general and Commercial Arithmetics in particular may be solved using the PSA.

The performance levels of the control groups have demonstrated the weaknesses of the conventional methods. In contrast to this, the PSA proved to be effective in promoting co-operative learning, which forms part of the solution to large classes in the context of inadequate human and material resources. The findings have shown that the PSA has the potential of encouraging high student participation in mathematics lessons and problem-solving activities. The PSA engendered social interactions that foster a sense of autonomous learning among students than the conventional methods used in this study. It also promotes conducive classroom environment that enhances the development and acquisition of mathematical concepts and skills and active student participation.

In essence, the PSA should be incorporated in mathematics teaching in Kenyan secondary schools. This will cause a significant improvement in students' achievement in KCSE mathematics examinations. In view of this, secondary school mathematics teachers are strongly encouraged to use this approach in their mathematics instruction.

6. CONCLUSIONS

The following conclusions have been drawn from the analysis of the data presented:

a) There were significant learning gains obtained by the students taught using PSA as compared to the low learning gains obtained by those students taught by conventional methods.

b) The PSA positively influenced the students' mathematics achievement that resulted in their autonomous learning and subsequent ownership of the lessons. Therefore, the PSA facilitates students' learning in mathematics better than the conventional teaching methods.

c) The difference in the achievements level is due to PSA, otherwise both groups have basic knowledge of mathematics.

7. RECOMMENDATIONS

On the basis of the findings of this study, the researcher made recommendations that the mathematics educators as well as education stakeholders can employ the PSA to enhance effective and efficient mathematics classroom interactions between the teachers and the students. These recommendations are: -

(i) PSA as a teaching strategy has beneficial influence on the achievement of secondary school mathematics students. Mathematics teachers should therefore enhance the use of the PSA teaching strategy to address the perennial problem of underachievement, especially among secondary school students.

(ii) Teacher Education curriculum developers should include the PSA in the training syllabus, thus making it part of the mathematics teacher education curriculum content.

(iii) The government should transform the textbooks of mathematics in problem based learning form, since the traditional textbooks do not meet the criteria of the PSA.

(iv) Extensive training programs, seminars and workshops should be organised for mathematics teachers in secondary schools to employ PSA in the classrooms. Of importance, the content of the PSA should be included in the regular in-service courses (e.g. Strengthening of Mathematics and Sciences in Secondary Education [SMASSE]) organised by the Ministry of Education for practicing teachers.

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Suggestions for Further Research:

The present study suggests that the PSA can be effective in improving mathematics instruction. However, there are areas that warrant further investigation such as the following:

a) More research should be conducted to test the influence of PSA as a teaching strategy on achievement, attitudes and motivation using other topics in mathematics other than the one used in the present study.

b) Investigation on the possible benefits derived from incorporating the use of computers in PSA need to be carried out.

c) A similar study on the influence of PSA on mathematics achievement should be carried out but using both qualitative and quantitative methods of data collections approach for concurrent triangulation and corroboration.

d) Systematic studies to determine the longevity or otherwise the influence of PSA on students' achievement, motivation and interest, among other variables.

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